CONFERENCE PROGRAMME AND ABSTRACT BOOK

13th International Conference on Ordered Statistical Data

22nd - 25th, May 2018 - Cádiz (Spain)

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH

ORGANIZING INSTITUTIONS:

UCA
Universidad de Cádiz

COLLABORATING INSTITUTIONS:

SEIO

OEA
GRUPO DE TRABAJO
ORDENACIONES ESTOCÁSTICAS
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Ayuntamiento de Cádiz

MINISTERIO
DE ECONOMÍA
Y COMPETITIVIDAD

INDESS
INSTITUTO UNIVERSITARIO DE INVESTIGACIÓN
PARA EL DESARROLLO SOCIAL SOSTENIBLE

Contents

Welcome to OSD2018

Continuing the series of Conferences in Mysore, India (2000), Warsaw, Poland $(2002?04)$, Izmir, Turkey (2005), Mashad, Iran (2006), Amman, Jordan (2007), Aachen, Germany (2008), Zagazig, Egypt (2010), Murcia, Spain (2012), Będlewo, Poland (2014), Piraeus, Greece (2016), the Statistic and Operational Research Department of the University of Cádiz will host the $13th$ **International Conference on Ordered Statistical Data OSD 2018**. The meeting will be held at Cádiz (Spain), $22nd - 25th$, May 2018 at the Conference Room of the Faculty of Nursing and Physiotherapy (a building of the University of Cádiz located in the main avenue of the town).

The conference will bring forth recent advances and trends in the mathematical theory of ordered statistical data, in order to facilitate the exchange of research ideas, promote collaboration among researchers from all over the world, and contribute to the further development of the field.

The meeting will be dedicated to all aspects of ordered statistical data, including:

- Approximations
- Bounds
- Characterisations
- Recurrence Relations
- Distribution Theory and Probability Models
- Stochastic Orders
- Reliability Theory and Survival Analysis
- Censoring
- Concomitants
- Statistical Interference
- Applications of Ordered Data
- Information and Entropies
- Nonparametric Methods
- Ranked Set Sampling
- Asymptotic Theory

Committees

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Miguel Ángel Sordo Díaz (Universidad de Cádiz)
Alfonso Suárez-Llorens (University of Cádiz, Spain)

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Miguel A. Sordo (University of Cádiz, Spain)
Alfonso Suárez-Llorens (University of Cádiz, Spain)

Conference Programme

Monday - 21^{st} May

18:00 - 21:00 Registration

Tuesday - 22^{nd} May

8:00 - 9:00 Registration

9:00 - 9:30 Opening Ceremony

9:30 - 10:30 Plenary talk
Chairman: **Haikady N. Nagaraja**

Keynote Speaker: **N. Balakrishnan**
Order Statistics and Symmetry

10:30 - 11:10 Session 1 - quantile estimation
Chairman: **N. Balakrishnan**

Speaker: **Chaitra H. Nagaraja**
A New Distribution-Free Method for Constructing Confidence Intervals for Quantiles

Speaker: **Haikady N. Nagaraja**
Large-Sample Properties of Jackknife Estimators of the Variance of a Sample Quantile

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11:10 - 11:40 Coffee Break

11:40 - 13:20 Session 2 - inference
Chairman: **Hon Keung Tony Ng**

Speaker: **Hanaa H. Abu-Zinadah**
Goodness-of-Fit Tests for The Exponentiated Gompertz Distribution

Speaker: **Magdy E. El-Adll**
A Novel Concept of a Stable Random Sample with Inference Based on Two-Parameter Exponential Generalized Order Statistics: Comparative Study

Speaker: **Marcus Johnen**
A likelihood equation for Weibull sequential order statistics

Speaker: **Manoj Chacko**
Inference on $P(X > Y)$ for bivariate normal distribution based on record values

Speaker: **Hon Keung Tony Ng**
Robust Parametric Estimation for Component Reliability based on System Lifetime Data

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13:20 - 13:30 Where is the restaurant?

13:30 - 15:30 Lunch

15:30 - 16:50 Session 3 - asymptotic theory
Chairman: **Haroom M. Barakat**

Speaker: **Artem Kovalevskii**
Asymptotics of an empirical bridge of a regression on concomitants

Speaker: **Anna Dembińska**
A strong limit law for proportions of near-maximum insurance claims under stationarity

Speaker: **El Sayed M. Nigm**
Asymptotic distributions of record values under exponential normalization stationarity

Speaker: **Haroom M. Barakat**
Asymptotic behavior of the maximum of multivariate order statistics in a norm sense

16:50 - 17:20 Coffee Break

17:20 - 18:00 Session 4 - ranked data sampling
Chairman: **Eugenia Stoimenova**

Speaker: **Nikolay I. Nikolov**
Mallows' models for imperfect rankings in ranked set sampling

Speaker: **Eugenia Stoimenova**
Distance-based models for partial rankings

18:00 Welcome Reception

Wednesday - 23^{rd} May

8:00 - 9:00 Registration

9:00 - 10:00 Plenary talk
Chairman: **Jorge Navarro**

Keynote Speaker: **Udo Kamps**
Order statistics with memory

10:00 - 11:00 Session 5 - distribution theory - characterizations
Chairman: **Mariusz Bieniek**

Speaker: **P. Yageen Thomas**
A Family of Random Variables Generated by a New Ordering Mechanism and Some of Its Applications

Speaker: **Magdalena Szymkowiak**
Generalized aging intensity

Speaker: **Mariusz Bieniek**
Uniqueness of characterization of discrete distributions by single regression of weak records

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11:00 - 11:30 Coffee Break

11:30 - 13:30 Session 6 - stochastic orders and reliability
Chairman: **Jafar Ahmadi**

Speaker: **Antonio Arriaza**
Stochastic comparisons of replacement policies in coherent systems under minimal repair

Speaker: **Camilla Calì**
Comparison results for inactivity times of systems under double monitoring

Speaker: **Maria Longobardi**
Coherent systems with dependent components: comparison results for inactivity times

Speaker: **Marco Burkschat**
On the dependence structure of failure-dependent lifetimes

Speaker: **Fatih Kızılaslan**
Stochastic comparisons of series and parallel systems with Gumbel distributed components

Speaker: **Jafar Ahmadi**
Stochastic orderings among general proportional mean past lifetime frailty models

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13:30 - 15:30 Lunch

15:30 - 16:30 Session 7 - progressive censoring - time varying stress models
Chairman: **Stefan Bedbur**

Speaker: **Alaa H. Abdel-Hamid**
Inference for a geometric-poisson-Rayleigh distribution under progressive-stress accelerated life tests based on type-I progressive hybrid censoring with binomial removals

Speaker: **Coşkun Kuş**
Statistical Inference for Kumaraswamy Distribution under Progressive Censoring Sample

Speaker: **Stefan Bedbur**
Testing the validity of a link function assumption in repeated type-II censored general step-stress experiments

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16:30 - 17:00 Coffee Break

17:30 - 20:30 Guided tour to explore the city of Cádiz

20:30 Cocktail dinner

Thursday - 24^{th} May

9:00 - 10:00 Plenary talk
Chairman: **Miguel A. Sordo**

Keynote Speaker: **Jorge Navarro**
Minimal repair of failed components in coherent systems

10:00 - 11:00 Session 8 - Advances in statistical modelling
Chairman: **Pepa Ramírez-Cobo**

Speaker: **Ignacio Cascos**
Process monitoring of a shifted exponential distribution through the origin-scale depth

Speaker: **Yoel G. Yera**
A bidimensional Markovian arrival process

Speaker: **Pepa Ramírez-Cobo**
Modeling the Operational Risk via Batch Markovian arrival processes

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11:00 - 11:30 Coffee Break

11:30 - 13:30 Session 9 - stochastic orders and applications
Chairman: **Félix Belzunce**

Speaker: **Miguel A. Sordo**
Stochastic orders and co-risk measures under positive dependence

Speaker: **Julio Mulero**
New stochastic comparisons based on tail values at risk

Speaker: **Alba M. Franco-Pereira**
A bootstrap algorithm for testing decreasing median residual life

Speaker: **Carolina Martínez-Riquelme**
Sufficient conditions for some transform orders

Speaker: **Gema Pigueiras**
A family of premium principles based on order statistics

Speaker: **Félix Belzunce**
A new preference type order for the stochastic dominance of dependent random variables

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13:30 - 15:30 Lunch

15:30 - 16:50 Session 10 - Bayesian inference
Chairman: **Inmaculada Barranco-Chamorro**

Speaker: **Alfonso Suárez-Llorens**
A class of prior applied to different premium principles

Speaker: **Marta Sánchez-Sánchez**
A bivariate class of priors applied to the estimation of the gamma distributions

Speaker: **Yee Lam Mo**
Bayesian analysis for the system lifetimes under Gumbel copulas of Weibull component lifetimes

Speaker: **Inmaculada Barranco-Chamorro**
Use of Linex loss function in progressively type-II censored samples

16:50 - 17:20 Coffee Break

17:20 - 18:20 Session 11 - distribution theory
Chairman: **Tomasz Rychlik**

Speaker: **Antonia Castaño-Martínez**
Exceedances of records

Speaker: **Andrzej Okolewski**
Extremal properties of order statistic distributions for samples with partially known multidimensional marginals

Speaker: **Tomasz Rychlik**
Upper and lower bounds on the variances of linear combinations of kth records

20:30 Gala dinner

Friday - 25^{th} May

10:00 - 11:00 Plenary talk
Chairman: **Alfonso Suárez-Llorens**

Keynote Speaker: **Fernando López-Blázquez**
Number of δ-records: exact and asymptotic distribution theory

11:00 - 11:30 Coffee Break

11:30 - 12:50 Session 12 - distribution theory - order statistics
Chairman: **Jacek Wesolowsky**

Speaker: **Ali İ. Genç**
Quotient of Order Statistics from Triangular Distribution

Speaker: **Zaher Abo-Eleneen**
On Some Applications of Cumulative Residual Entropy of Progressively Censored Order Statistics

Speaker: **Massoumeh Fashandi**
Some characterization results of symmetric distributions based on the properties of order statistics and records

Speaker: **Yousry H. Abdelkader**
Order statistics of Pearson III with applications to stochastics activity network

Speaker: **Jacek Wesolowsky**
Order statistics from overlapping samples: bivariate densities and regression properties

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12:50 - 13:30 Closing ceremony

13:30 - 15:30 Lunch

IMPORTANT INFORMATION:

- Registration, welcome reception and coffee breaks will be at the **Entrance Hall** of the Faculty of Nursing and Physiotherapy (University of Cádiz).
- All talks will be at the **Conference Room** of the Faculty of Nursing and Physiotherapy (University of Cádiz).
- The cocktail dinner will be at the Faculty of Economics and Business (University of Cádiz).
- All lunches will be at "Arteserrano" Restaurant.
- The Gala dinner will be at "Balandro" Restaurant.

Scan the QR code with your smartphone to see the location in a map.

Faculty of Nursing and Physiotherapy (University of Cádiz)
Avenida Ana de Viya, 52; 11009 Cádiz

Faculty of Economics and Business (University of Cádiz)
Avenida Enrique Villegas Vélez, 2; 11002 Cádiz

"Arteserrano" Restaurant
Paseo Marítimo, 2, 11010 Cádiz

"Balandro" Restaurant
Alameda de Apodaca, 22, 11004 Cádiz

Abstracts

Plenary Talks

Order Statistics and Symmetry

N. Balakrishnan (Plenary Talk)
(bala@mcmaster.ca, Department of Mathematics and Statistics, McMaster University)

In this talk, I will first describe some early developments on order statistics from symmetric distributions and highlight some of the important results, and some open problems that still exist. I will then describe a related quantile splicing method to produce some general skewed families of distributions with great flexibility. Finally, I will examine the reverse problem, namely, the symmetry of the underlying distribution based on the symmetry property between order statistics.

Order statistics with memory

Udo Kamps (Plenary Talk)
(udo.kamps@rwth-aachen.de, Institute of Statistics, RWTH Aachen University)

An extended model of order statistics based on possibly different distributions is introduced and examined. In the interpretation of successive failure times in a k-out-of-n system, say, where failures of components affect subsequent failure times, until each failure, the time periods under previous (increasing) loads exerted on the remaining components are recorded. Then the lifetime distribution of the system depends on the complete failure scheme. Thus, order statistics "with memory" provide an alternative to the model of sequential order statistics, which form a Markov chain. Some distributional properties of order statistics with memory and examples are shown. Relations between order statistics, sequential order statistics and order statistics with memory are pointed out, and respective spacings are compared by means of stochastic ordering.

Keywords: Order statistics, k-out-of-n system; Sequential order statistics; Spacings.

Reference

Katzur, A., Kamps, U. (2016). Order statistics with memory: A model with reliability applications. Journal of Applied Probability, 53 (4), 974-988.

Minimal repair of failed components in coherent systems

Jorge Navarro (Plenary Talk)
(jorgenav@um.es, Facultad de Matemáticas,Universidad de Murcia, Spain)
Antonio Arriaza
(antoniojesus.arriaza@uca.es, Dept. Statistics and Operations Research, University of Cádiz)
A. Suárez-Llorens
(alfonso.suarez@uca.es, Dept. Statistics and Operations Research, University of Cádiz)

The minimal repair replacement is a reasonable assumption in many practical systems. Under this assumption a failed component is replaced by another one whose reliability is the same as that of the component just before the failure (i.e. a used component with the same age). In this paper we study the minimal repair in coherent systems. We consider both the cases of independent and dependent components. Three replacement policies are studied. In the first one, the first failed component in the system is minimally repaired while, in the second one, we repair the component which causes the system failure. A new technique based on the relevation transform is used to compute the reliability of the systems obtained under these replacement policies. In the third case, we consider the replacement policy which assigns the minimal repair to a fixed component in the system. We compare these three options under different criteria and for different system structures. In particular we compare all the coherent systems with 1-4 independent and identically distributed components. Some general results are obtained as well under some assumptions. These results are included in the paper [1].

Keywords: Coherent systems; Minimal repair; Distorted distributions; Copula; Stochastic orders.

Bibliography:

[1] Navarro, J., Arriaza, A., Suárez-Llorens, A. 2018. *Minimal repair of failed components in coherent systems.* Submitted.

Number of δ-records: exact and asymptotic distribution theory

F. López-Blázquez (Plenary Talk)
(lopez@us.es, Departamento de Estadística e I.O., Universidad de Sevilla, Spain)

A δ-record is an observation at least δ units larger than the largest one already observed. Let $N_{n,\delta}$ be the number of δ-records in a random sample of size n from an absolutely continuous distribution. For $\delta \geq 0$, we discuss properties of $N_{n,\delta}$ and of the related statistic $N^*_{m,\delta}$, the number of δ-records among the first m ordinary records.
We present a number of results about the exact distributions of these statistics, and when $N_{\infty,\delta} = +\infty$ we present Laws of Large Numbers that provide the rate of growth of the number of δ-records, as well as normal and Poison approximations.
It may happen that under certain appropriate hypothesis $N_{\infty,\delta}$ is almost surely finite, but until now no explicit form of these limit distributions was known. Here we will present one example.

Keywords: Records; δ-records; Distribution theory; Laws of Large Numbers; Central Limit Theorem; Poisson approximations; q-calculus.

Contribution Talks

A New Distribution-Free Method for Constructing Confidence Intervals for Quantiles

Chaitra H. Nagaraja
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Haikady N. Nagaraja
(nagaraja.1@osu.edu, The Ohio State University)

Quantile estimation is important for a wide range of applications. Constructing confidence intervals around these estimates, however, is a more difficult problem because of the discrete nature of order statistics. Existing distribution-free techniques can be divided roughly into four categories: pivotal quantities, resampling, interpolation, and empirical likelihood methods. A new method based on pivotal quantities is introduced and a simulation study is used to compare performance across methods.

Large-Sample Properties of Jackknife Estimators of the Variance of a Sample Quantile

Haikady N. Nagaraja
(nagaraja.1@osu.edu, The Ohio State University, Division of Biostatistics, College of Public Health, 1841 Neil Avenue, Columbus, OH 43210 USA)
Chaitra H. Nagaraja
(cnagaraja@fordham.edu, Fordham University, Strategy and Statistics, Gabelli School of Business, 45 Columbus Avenue, New York, NY 10023 USA)

Using the joint limit distribution of adjacent spacings around an order statistic from a random sample of size n, we examine the limit properties of the family of delete-d jackknife estimators of the variance of a sample quantile for $d \geq 1$. We consider the central and intermediate order statistics and for the central case, we provide asymptotically unbiased delete-d jackknife estimators of its large-sample variance. For the sample median, we show that the limit distributions of the delete-d jackknife estimators of the variance

differ for sequences of odd and even values of $n-d$. In the intermediate case, the limit distribution of the delete-d jackknife estimator is free of d.

Keywords: Central order statistics; Intermediate order statistics; Quantile density function; Sample median; Jackknife; Convergence in distribution; Moment convergence.

Goodness-of-Fit Tests for The Exponentiated Gompertz Distribution

Hanaa H. Abu-Zinadah
(Department of Statistics, Sciences Faculty for Girls King Abdulaziz University, P. O. Box 32691, Jeddah 21438, Saudi Arabia)

Goodness-of-fit tests are considered for testing the three-parameters exponentiated Gompertz distribution based on complete and type II censored sampling and the maximum likelihood method of estimation are used for estimating the parameters. Critical values are obtained by Monte Carlo simulation for Kolmogorov-Smirnov, Anderson-Darling and Cramervon Mises type test statistics. A power study is carried out for these three test statistics for small, moderate and large censoring for different alternative models. Real data set will be used as an example for goodness-of-fit tests for the exponentiated Gompertz distribution.

Keywords: Goodness-of-fit tests; Exponentiated Gompertz distribution; Type II censoring; Maximum likelihood estimators; Monte Carlo simulation; Kolmogorov-Smirnov test; Anderson-Darling test; Cramer-von Mises test.

A Novel Concept of a Stable Random Sample with Inference Based on Two-Parameter Exponential Generalized Order Statistics: Comparative Study

Magdy E. El-Adll
(meladll2@hotmail.com, Department of Mathematics, Faculty of Science, Helwan University, Ain Helwan, Cairo, Egypt)

In order to explore the efficiency of a new statistical method, conducting simulation studies thousands of times for a given process is an indispensable procedure. In this paper a novel concept of a stable sample is presented, which can lead to new statistical methods, as well as saving time and effort substantially. Moreover, a comparative study based on Pitman closeness measure and mean square error are presented. Finally, some applications are given for illustrative purposes.

Keywords: Generalized order statistics; Pivotal quantity; Point predictor; Mean square error; Pitman's measure of closeness.

A likelihood equation for Weibull sequential order statistics

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Stefan Bedbur
(bedbur@isw.rwth-aachen.de, Institute of Statistics, RWTH Aachen University)
Udo Kamps
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In the literature, there are several examples for non-uniqueness of maximum likelihood estimators. Here, a multi-sample set-up of sequential order statistics from Weibull distribution functions with a common unknown shape parameter is considered. The respective likelihood equation may have multiple roots even in the single-sample case, which is demonstrated by a simple example and illustrated with a simulation study. Uniqueness of the maximum likelihood estimator is examined with respect to different models of ordered data, sufficient conditions for uniqueness are shown, and a known algorithm is applied to determine the MLE in case of multiple roots.

Keywords: Maximum likelihood estimation; Order statistics; Sequential order statistics; Weibull distribution.

Bibliography:

[1] Barnett, V.D., 1966. Evaluation of the maximum-likelihood estimator where the likelihood equation has multiple roots. *Biometrika 53(1/2)*, 151–165.

[2] Cramer, E., Kamps, U., 1996. Sequential order statistics and k-out-of-n systems with sequentially adjusted failure rates. *Annals of the Institute of Statistical Mathematics 48(3)*, 535–549.

[3] Kavvadias, D.J., Vrahatis, M.N., 1996. Locating and computing all the simple roots and extrema of a function, *SIAM Journal on Scientific Computing 17(5)*, 1232–1248.

Inference on $P(X > Y)$ for bivariate normal distribution based on record values

Manoj Chacko
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In this paper, we consider the problem of estimation of $R = P(X > Y)$, when (X, Y) follows bivariate normal distribution. The Maximum likelihood estimates (MLEs) and Bayes estimates (BEs) of R are obtained based on record values and its concomitants. BEs are obtained based on both symmetric and asymmetric loss functions. The approximate, bootstrap and credible confidence intervals for R are also obtained. Monte Carlo simulations are carried out to study the accuracy of the proposed estimators.

Keywords: Record values; Bivariate normal distribution; Bayesian estimation.

Robust Parametric Estimation for Component Reliability based on System Lifetime Data

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Ping Shing Chan
(benchan@cuhk.edu.hk, Department of Statistics, The Chinese University of Hong Kong)

In this paper, we consider the estimation of lifetime characteristics of the components in a system based on system lifetime data with known system structure using minimum density divergence estimation method. Based on the features of system lifetime data, different minimum density divergence estimators are proposed for complete and censored system lifetime data. A Monte Carlo simulation study is used to evaluate the performance of those proposed estimation procedures and compare with the maximum likelihood estimation method under different contaminated models. The simulation results showed that those proposed estimation procedures are more robust compared to the maximum likelihood estimation method under contaminated models. A numerical example is used to illustrate the methodologies. Finally, some on-going research work and future research directions are discussed.

Keywords: Likelihood inference; Minimum density divergence estimator; Right censoring.

Asymptotics of an empirical bridge of a regression on concomitants

Artem Kovalevskii
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The research was supported by RFBR grant 17-01-00683)

We propose a class of tests for linear regression on concomitants (induced order statistics). These tests are based on sequential sums of regression residuals. The sums form an empirical bridge of the regression model by self-centering and self-normalising. We prove weak convergence of the empirical bridge in uniform metrics to a centered Gaussian process. The tests are of chi-squared type.

Keywords: Concomitants; weak convergence; Regression residuals; Empirical bridge.

A strong limit law for proportions of near-maximum insurance claims under stationarity

Anna Dembińska
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Let a sequence of observations $(X_n, n \geq 1)$ form a strictly stationary process. By $X_{1:n}$, $X_{2:n}, \ldots, X_{n:n}$ denote the order statistics corresponding to the sample $(X_1, X_2, \ldots, X_n)$. Following the standard notation, we say that $(X_{k_n:n}, n \geq 1)$ is a sequence of (1) *extreme order statistics* if and only if (iff) k_n or $n-k_n$ is fixed; and (2) *intermediate order statistics* iff $\min(k_n, n-k_n) \to \infty$ and $k_n/n \to \lambda \in \{0, 1\}$ as $n \to \infty$.
In this talk we will study the asymptotic behavior of proportions of observations in the sample that fall near extreme or intermediate order statistic $X_{k_n:n}$,

$$P_n(a) := \frac{\#\{j \in \{1, \ldots, n\};\ X_j \in (X_{k_n:n} - a, X_{k_n:n})\}}{n},$$

as $n \to \infty$, where $a > 0$. We will show that for any $a > 0$ the random sequence $(P_n(a), n \geq 1)$ converges almost surely to some random variable. We will also describe the law of the limiting variate. Finally, we will apply the presented results in insurance to examine the long-term behaviour of the number of near-maximum insurance claims.
The talk will be based on my joint paper with A. Buraczyńska [1].

Keywords: Almost sure convergence; Extreme and intermediate order statistics; Near-maximum insurance claims; Stationary processes.

Bibliography:

[1] Buraczyńska, A., Dembińska, A. (2018). The long-term behaviour of the number of near-maximum insurance claims. *Manuscript*.

Asymptotic Distributions of Record Values under Exponential Normalization

E.M. Nigm
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In this paper, we study the limit distribution of the record values under nonlinear normalization of the form $\exp\{u_n(|\log|x||)^{v_n}\text{sign}(\log|x|)\}\text{sign}(x)$, which is called exponential norming. The corresponding limit laws of the upper extremes are called e-max stable laws (denoted by $U(.)$). In this paper, we show that the limit distributions of the record values under exponential norming are of the form $\mathcal{N}(-\log(-\log U(x)))$, where $\mathcal{N}(.)$ is the standard normal distribution. Moreover, we study the domains of attraction for these types of limit laws. Finally, some illustrative examples are given.

Asymptotic behavior of the maximum of multivariate order statistics in a norm sense

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In this work we investigate the asymptotic behavior of the extremes of a bivariate data by using the Reduced Ordering Principle (R-ordering). When, the sup-norm is used, we reveal the interrelation between the R-ordering principle and Marginal Ordering Principle (M-ordering). The asymptotic behavior of the maximum sup-norms corresponding to the bivariate data is completely determined.

Keywords: Weak convergence; Multivariate extremes; Reduced Ordering Principle; Marginal Ordering Principle; Sup-norm; D-norm.

Mallows' models for imperfect rankings in ranked set sampling

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Eugenia Stoimenova
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Let $\left\{X_{[1]}, X_{[2]}, \ldots, X_{[k]}\right\}$ be a *one-cycle balanced ranked set sample* obtained from k random samples (sets) of size k. If the ranking within each random sample (set) is correct, we obtain *perfect ranking*. However, since the ranking is not based on actual measurement, it may be inaccurate and contain errors. In this case we will have *imperfect ranking*. Our interest is in testing the hypothesis of perfect ranking versus the general alternative of imperfect rankings.
By arranging the observations $\left\{X_{[1]}, X_{[2]}, \ldots, X_{[k]}\right\}$ in an increasing order, we obtain the *ordered ranked set sample* (ORSS)

$$X_{1:k}^{ORSS} \leq X_{2:k}^{ORSS} \leq \ldots X_{k:k}^{ORSS},$$

introduced by Balakrishnan and Li [1]. Li and Balakrishnan [4] proposed three non-parametric tests for perfect ranking based on ORSS and metrics on permutations. We introduce new similar tests and compare them to the existing tests in the literature. In order to compare the power of these tests, one should first consider an alternative model for imperfect rankings. Some of the most commonly used models are: Bivariate normal model proposed by Dell and Clutter [2], Fraction of random rankings by Frey et al. [3], Fraction of inverse rankings by Frey et al. [3] and Fraction of neighbor rankings by Vock and Balakrishnan [6]. Since the parameters of these four models can not be estimated directly, the power comparisons could be made only via simulation studies. As an alternative, we suggest using the Mallows' models for imperfect rankings.
The probability mass function of the Mallows' models [5] is given by

$$P_{\theta,\pi_0}(\pi) = e^{\theta d(\pi,\pi_0) - \psi_k} \quad \text{for} \pi \in \mathcal{S}_k,$$

where θ is a real parameter ($\theta \in \mathbb{R}$), $d(\cdot,\cdot)$ is a metric on the permutation group $\mathcal{S}_k$, π_0 is a fixed ranking and ψ_k is a normalizing constant. Since the judgment ranking is expected to be close to the perfect ranking, we can assume that π_0 is the identity $e_k \in \mathcal{S}_k$ and $\theta \leq 0$. If we fix the metric $d(\cdot,\cdot)$, then ψ_k can be transformed into a function of θ. Therefore, θ can be considered as the only unknown parameter of the Mallows' model. It is not possible to find the maximum likelihood estimate of θ directly. However, we make use of the Expectation-Maximization (EM) algorithm and compare the power of several tests under the Mallows' alternative based on different metrics on $\mathcal{S}_k$.

Keywords: Ordered ranked set sample; Imperfect ranking; Power comparisons; Mallows' models .

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Distance-based models for partial rankings

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In many situations, there are different methods for analyzing the same data. For example, several methods exist for finding differentially expressed genes using RNA-seq data. They tend to produce similar, but not identical significant genes and rankings of the gene list. When comparing different methods applied to the same data, we are interested in how close are their outputs. The main idea is to define appropriate distance of the sample space.
In this paper we define an appropriate mathematical framework that include special cases of partially ranked lists of items. Any ranked list can be complete, which means all n items are ranked, or incomplete, which means some items are not ranked. The incomplete ranking include the case where the most significant k items are ranked, with group $k+1$ consisting of the remaining items. Any complete ranking of n items corresponds a permutation $\langle \alpha(1), \ldots, \alpha(n) \rangle$ from the set of all permutations S_n.
Given a set of rankings, the problem of their comparison reduced to a problem of choosing appropriate measure of association on the set of of permutations S_n. We define appropriate distance measures on S_n in order to compare complete or incomplete rankings. The distance can be thought of as a measure of the similarity of the two rankings. Our aim here is to compare how similar/dissimilar two incomplete rankings are based on the number of items present in the same ordered groups in both rankings. We consider extensions of some popular metrics on permutations to include partial rankings. These include Kendall's τ, Spearman's ρ, Spearman's footrule and few more $[2,\ 1,\ 3]$.

Keywords: Partial rankings, Distances on partial rankings.

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A Family of Random Variables Generated by a New Ordering Mechanism and Some of Its Applications

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This paper describes an ordering principle by which the observations are rearranged into a new system of random variables. The marginal and joint distributions of these random variables are derived. It is shown that except for the extreme member random variables in the new system, all others are distributed bimodally. The graphs of the generated probability density functions of these random variables are shown to be two piece density types with a different mode for each of the two pieces. This knowledge of obtaining bimodal distributions has been further applied to generate a more general bimodal family of distributions. Some real life data sets for which the member distributions in the generated generalized class of bimodal distributions found better fit for them have been identified and illustrated. The above technique of generating bimodal distributions has been further extended to generate trimodal, quadrimodal and many more plurimodal distributions.

Keywords: Bimodal Distributions; Plurimodal Distributions; Ordered Random Variables; Modelling of Two Peaks Data.

Generalized aging intensity

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We introduce and study a family of generalized aging intensity functions. The functions characterize distributions of univariate positive absolutely continuous random variables. Further on, we define and analyze the generalized aging intensity orders.

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Uniqueness of characterization of discrete distributions by single regression of weak records

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Let $\{W_n,\, n \geq 0\}$ denote the sequence of weak records of the sequence of independent identically distributed random variables with common distribution F with the support on the set $\mathbb{N}$ of non-negative integers. Fix $r \geq 0$ and $\ell \geq 1$ and let $h : \mathbb{N} \to \mathbb{R}$ be any strictly increasing function. We consider the problem of the unique identification of F by the knowledge of single regression function

$$\xi(j) = \mathbb{E}\left(h\left(W_{r+\ell}\right) \mid W_r = j\right), \quad j \in S.$$

We show that this function uniquely determines the underlying distribution F if and only if the corresponding system of $\ell - 1$ difference equations has the unique solution. This result is applied to obtain new proofs of known results as well as new characterizations of discrete distributions.

Stochastic comparisons of replacement policies in coherent systems under minimal repair

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In this work we study stochastic comparisons of coherent systems under different replacement policies. We consider minimal repair in the components, that is, a failed component is replaced by a component which is similar (in law) to the replaced one and with the same age. Under these assumptions we study how to determine the best replacement policies. Two general methods are proposed, one for systems with identically distributed components and another one for systems with ordered components. Both can be applied to independent or dependent components. In particular, we determine the optimal replacement policy in the cases of series and parallel systems with independent components. Other examples are studied as well. These results are included in the paper [1].

Keywords: Coherent systems; Minimal repair; Stochastic orders; Distorted distributions.

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Comparison results for inactivity times of systems under double monitoring

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Let T be the system lifetime, the inactivity time of the system at t is $(t-T|T<t)$. Under double inspections, we typically may know that the system was working at a time t_1 but that is broken at another time $t_2 > t_1$. Then we obtain representations for the reliability function of inactivity time of the system, that is $(t_2 - T|t_1 < T < t_2)$, through distortion functions considering both the cases of independent and dependent components.

Keywords: Coherent system; Inactivity time; Distorted distributions.

Coherent systems with dependent components: comparison results for inactivity times

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Consider a coherent system (having lifetime T) with possibly dependent components, and assume that it failed before a given time $t > 0$. Its inactivity time $t-T$ can be evaluated under different conditional events. In fact, one might just know that the system has failed and then consider the inactivity time $(t-T|T \leq t)$, or one may know which ones of the components have failed before time t, and then consider the corresponding system's inactivity time under this condition. For all these cases we obtain a representation of the reliability function of system inactivity time and study new stochastic comparison results for inactivity times under the different conditional events.

Keywords: Reliability; Inactivity time; Coherent systems; Stochastic orders; Distortion functions; Copula.

On the dependence structure of failure-dependent lifetimes

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The model of sequential order statistics has been proposed to describe ordered failure times of components in a system. After every failure of a unit, the underlying lifetime distribution of the remaining components may change, e.g., for reflecting effects of additional load. The unordered failure times of the units in the system, the failure-dependent component lifetimes, provide a dependence model that allows component failures to affect the performance of intact components. In the talk, dependence properties of the failure-dependent lifetimes are presented.
The talk is based on joint work with E. Bezgina.

Stochastic comparisons of series and parallel systems with Gumbel distributed components

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In this study, stochastic comparisons of series and parallel systems having Gumbel distributed components are studied. These comparisons are investigated with respect to usual stochastic order, hazard rate order, reversed hazard rate and likelihood ratio order. Extreme value theory deals with the stochastic behavior of the extreme values in a process. The maximum and the minium of a random sample after proper randomization can only converge in distribution to one of the three possible distributions, the Gumbel distribution, the Frèchet distribution or the Weibull distribution. Extreme values distributions are

widely used in finance, insurance, economics, hydrology, meteorology, engineering, material sciences, telecommunications, and many other industries dealing with extreme events. A random variable X is said to have Gumbel distribution, if its cumulative distribution function is

$$F(x;\mu,\sigma) = \exp\left\{-\exp\left(\frac{x-\mu}{\sigma}\right)\right\}, \ -\infty < x < \infty, \ -\infty < \mu < \infty, \ \sigma > 0,$$

where μ and σ are the location and scale parameters, respectively. Gumbel distribution is also called type I extreme value distribution. The Gumbel distribution was introduced by Gumbel [2] and since then it received a considerable attention in the literature. It is frequently used in engineering and climate modeling. It is known that Gumbel distribution is the limiting distributions of the largest $(X_{n:n})$ and the smallest $(X_{1:n})$ order statistics of some well known lifetime distributions. The limiting distribution of $X_{n:n}$ is Gumbel when $X_i, i = 1, ..., n$ are normal, log-normal, exponential, Gamma, Laplace, logistic, Rayleigh distribution and limiting distribution of $X_{1:n}$ is also Gumbel when $X_i, i = 1, ..., n$ are normal, log-normal, Laplace, logistic distribution see Table 3.1 in [3] and Tables 3.2 and 3.3 in [1]. Series and parallel systems play an important role in reliability. Since the Gumbel distribution is the limiting distributions of series and parallel systems for some well known lifetime distributions, stochastic comparisons of these systems having Gumbel distributed components will be important. In this study, comparison the lifetimes of the series and parallel systems are considered in terms of the usual stochastic order, hazard rate order, reversed hazard rate and likelihood ratio order. Our results are obtained under the different assumptions for the parameters.

Keywords: Gumbel (Type I extreme value) distribution; Parallel and series systems; Usual stochastic order; Hazard rate order; Reversed hazard rate order; Likelihod ratio order.

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Stochastic orderings among general proportional mean past lifetime frailty models

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There are several well-known models for modeling and analyzing failure time data, namely, the proportional hazard (PH) rate model, the proportional reversed hazard (PRH) rate model, the additive hazard model, the additive reversed hazard rate model, proportional mean residual life model and frailty model. In this paper, the general proportional mean past lifetime (GPMPL) frailty model is considered. Stochastic comparisons are made through which it has been shown that some well known orderings between two frailty random variables carry over to the corresponding lifetime variables. Some comparisons between two frailty models are made.

Keywords: Frailty model; Mean past lifetime; Proportional hazard rate model; Stochastic ordering.

Inference for a Geometric-Poisson-Rayleigh Distribution Under Progressive-Stress Accelerated Life Tests Based on Type-I Progressive Hybrid Censoring with Binomial Removals

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Based on failures of a parallel-series system, a new distribution called geometric-Poisson-Rayleigh distribution is proposed. Several properties of the distribution are discussed. A real data set is used to compare the new distribution with other six distributions. The progressive-stress accelerated life tests are considered when the lifetime of an item under use condition is assumed to follow the geometric- Poisson-Rayleigh distribution. It is assumed that the scale parameter of the geometric-Poisson-Rayleigh distribution satisfies the inverse power law such that the stress is a non-linear increasing function of time and the cumulative exposure model for the effect of changing stress holds. Based on type-I progressive hybrid censoring with binomial removals, the maximum likelihood and Bayes (using linear-exponential and general entropy loss functions) estimation methods are considered to estimate the involved parameters. The Bayes estimates are obtained using Markov chain Monte Carlo algorithm. Finally, a simulation study is performed and numerical computations are carried out to compare the performance of the implemented estimation methods.

Keywords: Parallel-series system; Progressive-stress accelerated life test; Progressive hybrid censoring; Binomial removals; Maximum likelihood and Bayes estimations; Simulation.

Statistical Inference for Kumaraswamy Distribution under Progressive Censoring Sample

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The APE distribution is introduced by Mahdavi and Kundu (2017) and they studied statistical inference for the parameters based on complete data. In this presentation, the point and interval estimation of APE parameters are considered. The maximum likelihood estimates, exact and asymptotic confidence intervals of parameters are discussed under progressive censoring. Simulation study is performed to investigete the performance of estimates and intervals.

Keywords: Kumaraswamy distribution; Confidence interval; Maximum likelihood estimation; Progressive censoring.

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Testing the validity of a link function assumption in repeated type-II censored general step-stress experiments

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In step-stress experiments, test units are successively exposed to higher usually increasing levels of stress to cause earlier failures and to shorten the duration of the experiment. When parameters are associated with the stress levels, one problem is to estimate the parameter corresponding to normal operating conditions based on failure data obtained under higher stress levels. For this purpose, a link function connecting parameters and stress levels is usually assumed, the validity of which is often at the discretion of the experimenter. In a general step-stress model based on multiple samples of sequential order statistics, we provide exact statistical tests to decide whether the assumption of some link function is adequate. The null hypothesis of a proportional, linear, power or log-linear link function is considered in detail, and associated inferential results are stated. In any case, except for the linear link function, the test statistics derived are shown to have only one distribution under the null hypothesis, which simplifies the computation of (exact) critical values. Asymptotic results are addressed, and a power study is performed for testing on a log-linear link function. Some improvements of the tests in terms of power are discussed.

Keywords: Accelerated life testing; Step-stress model; Link function; Sequential order statistics; Maximum likelihood estimation; Hypothesis testing.

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Process monitoring of a shifted exponential distribution through the origin-scale depth

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A new notion of origin and scale depth is introduced and its application in the joint monitoring of the parameters of a shifted exponential process is presented. The control chart built out of the origin-scale depth here introduced is compared with other control charts previously presented in the statistical literature in terms of their Average Run Lengths.

Keywords: Joint Monitoring; Origin and Scale; Parameter Depth.

Introduction

We use the parameter depth framework introduced in [1] in order to build a notion of origin-scale depth. The joint monitoring of processes governed by their origin and scale parameters, such as the shifted exponential one, is performed through control charts based on a parameter depths as described in [2]. Finally, the performance of our proposal is compared with the one of the control charts analysed in [3].

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A bidimensional Markovian arrival process

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With the purpose of modeling a real bidimensional dataset in the context of reliability, in this work we present an extension to the two-dimensional case of the Markovian arrival process (MAP). The novel process keeps the properties of Markovian arrival traces in marginal way, but also allows for dependence between the inter-event times sequences. The Marshall-Olking exponential distribution, the point of departure for the definition of the model, will be reviewed in this talk, where in addition, some indications related to the statistical inference for the novel model will be provided.

Keywords: Markovian arrival process (MAP); Bidimensional process; Marshall-Olking exponential distribution.

Modeling the Operational Risk via Batch Markovian arrival processes

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Because of its high expected impact to the banking industry, since the 9/11 attacks and tradings at Société Général, AIB and National Australia Bank, there is an interest in properly modeling the Operational Risk (OpRisk). In this work we propose a model for the OpRisk based on the Batch Markovian Arrival Process (BMAP), a general class of point processes for which the inter-losses times are correlated and distributed according a phase

type distribution. Under the classic loss distribution approach (LDA), the frequency is modeled by a specific type of BMAP, and the severity is estimated using a double Pareto Lognormal distribution. Numerical results with a real dataset related to operational risk will be presented to illustrate the performance of the novel approach.

Keywords: Operational Risk; Loss modeling; Dependent losses times; Point process; Batch Markovian arrival process; PH distribution; Double-Pareto Lognormal distribution.

Stochastic orders and Co-risk measures under positive dependence

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Conditional risk measures (or Co-risk measures) and risk contribution measures are increasingly used in actuarial portfolio analysis to evaluate the systemic risk, which is related to the risk that the failure or loss of a component spreads to another component or even to the whole portfolio: while Co-risk measures are risk-adjusted versions of measures usually employed to assess isolate risks, risk contribution measures quantify how a stress situation for a component affects another one. In this paper, we provide sufficient conditions under which two random vectors could be compared in terms of CoVaR (conditional value-atrisk), CoES (conditional expected shortfall) and different risk contribution measures. Conditions are given in terms of the increasing convex order, the dispersive order and the excess wealth order of the marginals under some assumptions of positive dependence.

Keywords: Co-risk measures; Stochastic orderings; CoVaR; CoES; Risk contribution.

New stochastic comparisons based on tail values at risk

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In this work, we provide a new criteria for the comparison of claims, when we have conditional claims arising in stop loss contracts or contracts with franchise deductible. These stochastic comparisons are made on the basis of the tail value at risk (also known as conditional tail expectation), just for a fixed level and beyond. In this work, we study the interest of comparing these quantities, preservation properties and in addition, we provide sufficient conditions for its study and illustrate its usefulness with some examples.

Keywords: Value at risk; Tail value at risk; Residual claims.

A bootstrap algorithm for testing decreasing median residual life

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In applications it is common to categorize life distributions according to different aging properties based on different reliability measures. In this work we propose a test statistic to check the aging notion known as *decreasing percentile residual life*. This estimator is based on the difference of the empirical percentile residual life function and the monotone estimator. A simulation study has been carried out to see the performance of the test.

Keywords: Residual life; Bootstrap; Aging.

Sufficient conditions for some transform orders

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In this talk we present several results that relate the unimodality of the ratio of two quantile density functions with some transform orders. In particular, we provide sufficient and, in some cases, necessary conditions for the star-shaped, qmit and dmrl orders. These results are intended to be a tool for the comparison in the previous orders, when the convex order does not hold. Additional results about implications in the context of ageing notions are given.

Keywords: Star-shaped; qmit and dmrl orders; Sufficient conditions.

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A family of premium principles based on order statistics

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In this paper, we further investigate a family of distortion premium principles based on mixtures of tail value-at-risks, with beta mixing distributions, introduced by Sordo et al. (2016). We show that each premium principle of this family can be represented in terms of the expected average risk of the largest claims in a set of independent and identically distributed claims. From this representation, we obtain a convergence result that interprets the tail value-at-risk at level p of X as the arithmetic average of the 100(1-p) percent largest claims of a large enough number of independent claims with the same distribution as X. Characterizations of the stop-loss order and the excess-wealth order in terms of this family of premiums are provided. As a consequence, we obtain sufficient conditions for ordering the net premiums of two collective risks under the LCR and ECOMOR reinsurance treaties.

Keywords: Premium principle; Order statistics; Distortion function; Stop-loss order; Excess-wealth order; Reinsurance.

A new preference type order for the stochastic dominance of dependent random variables

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The usual stochastic order is a tool to compare the magnitude of two random variables. However, this criteria only takes into account the marginal distributions of the two random variables, and does not take into account their possible dependence. An alternative, in the dependent case, is the precedence order, but this criteria is not very informative, given that it reduces all the information of the bivariate random vector in just two numbers. In this talk, we present from an applied point of view, a new criteria of stochastic dominance that takes into account the dependence structure of the two random variables involved in the comparison. Relationships with some existing criteria, closure properties and applications are also given.

Keywords: Joint stochastic orders; Preference order.

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A class of priors applied to different premium principles

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In the context of robust Bayesian analysis, we focus on a new class of prior distributions based on stochastic orders and distortion functions defined in Arias-Nicolás et al. (2016). The problem of computing most usual premium principles in risk theory will be analyzed. We will consider that uncertainty with regard to the prior distribution can be represented by the assumption that the unknown prior distribution belongs to the new class of distributions and we will examine the ranges of the Bayesian premium when the priors belong to such a class. Finally, some applications are shown.

Keywords: Robustness Bayesian Analysis; Prior class; Stochastic orders; Distortion functions; Premiums.

Bibliography:

[1] Arias-Nicolás, J.P., Ruggeri, F. and Suárez-Llorens, A. (2016). New classes of priors based on stochastic orders and distortion functions. *Bayesian Analysis*, 11, **4**, pp. 1107-1136

A bivariate class of priors applied to the estimation of the gamma distribution

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In the context of robust Bayesian analysis, we find in Arias-Nicolás et al. (2016) a class of prior distributions based on stochastic orders and distortion functions. An extension to the bivariate problem will be analyzed in this work. We will consider that uncertainty in the bivariate prior distribution can be represented by using a bivariate class of priors based on stochastic orders and weight functions. We analyze the ranges of the bivariate posterior distributions when the priors belong to such a class. Additionally, we will show some results related to the estimation of the scale and shape parameters of the gamma distribution. Finally, we will present some numerical results.

Keywords: Robustness Bayesian Analysis; Prior class; Stochastic orders; Weight functions; Multivariate distributions.

Bibliography:

[1] Arias-Nicolás, J.P., Ruggeri, F. and Suárez-Llorens, A. (2016). New classes of priors based on stochastic orders and distortion functions. *Bayesian Analysis*, 11, **4**, pp. 1107-1136

Bayesian analysis for the system lifetimes under Gumbel copulas of Weibull component lifetimes

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Weibull distribution have been used widely in modelling failure data and reliability problems because of its wide range of shapes in density functions. In this paper, a coherent system of n Weibull lifetime components is considered. We assume the joint distribution of these n identically distributed components after installed into the system satisfies Gumbel copulas. Given a random sample of m system lifetimes we discuss the Bayesian analysis of the unknown parameters. Posterior distribution of the unknown parameters is obtained using MCMC methods. Numerical simulations will be presented.

Keywords: Bayesian Analysis; Gumbel Copulas; System Signatures; Reliability; MCMC algorithm.

Use of Linex loss function in progressively type-II censored samples

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The problem of estimation of the shape parameter in a generalized half-logistic distribution for progressively type-II censored samples is of interest in reliability and survival analysis. In this paper, Bayesian methods of estimation based on quadratic and Linex loss functions are proposed. Closed expressions and approximations are obtained for the Bayes and posterior risks of Bayes estimators. These results allow us to asses the performance of estimators obtained under the previously cited loss functions. An application to a real data set and a simulation study are included where the importance of the different features involved in the progressive type-II censoring scheme is also shown.

Keywords:Generalized half-logistic distribution; Progressive censoring; Linex loss; Bayes risk; Burr-XII model.

Exceedances of records

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Given a sequence of random variables (rv's) and a real function ψ, the ψ-exceedances form a subsequence consisting of those rv's larger than a function, ψ, of the previous element of the subsequence. We present the basic distribution theory of ψ-exceedances for a sequence of independent and identically distributed rv's. We give several examples and we study with more detail the case of exponential parents with ψ a linear function. The particular case of arithmetic exceedances is useful to describe the behavior of a type I counter when the arrival process of particles follows a non-homogeneous Poisson process. We also mention applications to destructive testing, early alert systems and the departure process of a $M_t/D/1/1$ queue.

Keywords: Records; Record-like random variables; Exceedances; Extreme value theory; Counters of particles; Counting processes.

Extremal properties of order statistic distributions for samples with partially known multidimensional marginals

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Let $X = (X_1, \ldots, X_n)$ be an n-tuple of random variables where each X_j has the same known distribution function F and where there is a number $k \leq n$ such that for each $i \in \{1, \ldots, k\}$, all i-tuples have copulas with the same known diagonal δ_i. A reliability system with such nonnegative component lifetimes $X_1, \ldots, X_n$ is a system with the property that for each $i \leq k$, all of its structurally identical sub-systems of i components have the same known reliability function. We provide a characterization for empirical distributions from the X_j's, and apply it to derive two-sided bounds (depending on F and δ_i's) for arbitrary linear combinations of distribution functions of the associated order statistics as well as to establish necessary and sufficient conditions for uniform sharpness of these bounds. Moreover, for $k = 2$ and some classes of δ_2's, we determine stochastically extremal distributions of single order statistics.

Keywords: Order statistics; Dependent observations; Copula; Diagonal section; Empirical distribution function; Coherent systems.

Upper and lower bounds on the variances of linear combinations of kth records

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We describe a method of determining upper bounds on the variances of linear combinations of kth records values from i.i.d. sequences, expressed in terms of variances of parent distributions. We also present conditions for which the bounds are sharp, and those for which the respective lower ones are equal to zero. General results are specified in the special case of kth record spacings, i.e. the differences of consecutive kth record values.

Keywords: kth record value; Linear combination; kth record spacing; Variance, Sharp bound.

Quotient of Order Statistics from Triangular Distribution

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Exact expression for the distribution of the quotient of any two order statistics from the triangular distribution is obtained. Especially, the extremal quotient which is defined as the ratio of the sample maximum to the sample minimum is studied.

Keywords: Extremal quotient; Quotient of order statistics; Triangular distribution.

On Some Applications of Cumulative Residual Entropy of Progressively Censored Order Statistics

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The cumulative residual entropy (CRE) is a measure of information proposed by Rao et al. (2004, IEEE Transactions on Information Theory, 50, $1220-1228$). In this paper, we express the joint CRE of progressively Type-II censored order statistics in terms of a single integral. Some useful recurrence relations for the CRE in progressively Type-II censored order statistics are derived. The results developed here provide simple computational formulas for the CRE in progressively censored order statistics. Then we use the joint CRE of progressively Type-II censored order statistics for characterizing the parent distribution. We also develop a nonparametric estimation method for the CRE based on the observed

progressively censored sample.

Keywords: Characterization; Entropy; Information measure; Nonparametric estimation; Recurrence relations.

Some characterization results of symmetric distributions based on the properties of order statistics and records

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It is known that the class of univariate symmetric distributions is so broad and includes several well-known distributions such as normal, logistic, student-t, Cauchy, Laplace and uniform distributions. In this paper, using the completeness properties of the sequence of functions, we establish some characterization for univariate symmetric distribution based on certain properties. The results are based on the identity in distribution, equality of the moments and Kerridge measure of inaccuracy of the lower and upper order statistics as well as upper and lower k-records. Also, for Farlie-Gumbel-Morgenstern (FGM) family, we obtain some characterization results for univariate symmetric distributions based on the concomitants of upper and lower order statistics as well as concomitants of upper and lower k-records.

Keywords: Bivariate FGM distribution; Completeness property; Concomitant; Kerridge measure of inaccuracy; Order statistics; k-records; Symmetric Distribution.

Order statistics of Pearson III with applications to stochastic activity networks

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Explicit expressions are derived for moments of order statistics arising from independent and nonidentically distributed Pearson III random variables. Applications to reliability analysis and stochastic activity networks are given. Mathematica7 codes to perform the calculations are also given.

Keywords: Order Statistics; Moments; Pearson III distribution; Reliability; Project Planning.

Order statistics from overlapping samples: bivariate densities and regression properties

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We are interested in joint distribution of two order statistics from overlapping samples. We give an explicit formula for the distribution of such a pair of random variables under the assumption that the parent distribution is absolutely continuous (with respect to the Lebesgue measure on the real line). The distribution is identified through the form of the density with respect to a proper combination of the bivariate Lebesgue measure on R^2 and the univariate Lebesgue measure on the diagonal $\{(x, x) : x \in R\}$.
We are also interested in the question to what extent conditional expectation of one of such order statistic given the other determines the parent distribution. In particular, we provide a new characterization by linearity of regression on an order statistics from the extended sample given the one from the restricted sample which solves a problem explicitly stated in the literature. It appears that, to describe correct parent distribution in many cases, it is convenient to use quantile density functions. In several such cases we provide new results regarding uniqueness of regressions of order statistic. Nevertheless the general question of identifiability of the parent distribution by regression for order statistics from overlapping samples remains open.

Participants in OSD2018